Applications

- Optical multiplexing
- High power applications
- Optical Pump for Amplifier
- PON

Overview

Optical power in two or more waveguides can be combined onto a single waveguide. This note describes general properties of a symmetric optical combiner, and shows simulations by OptiBPM. It is possible to achieve lossless operation, such that all the optical power at the inputs reaches the output. However, it turns out that lossless operation is only possible for specific input conditions. For example, if one of the inputs goes dark, there will be an inevitable 3 dB loss on the other arm. This note shows this surprising result in two ways: by theoretical means, and by detailed simulation of the optical propagation by OptiBPM.

Theory

The waves are characterized by optical phasors, and may have arbitrary amplitude and phase. The normalization is according to power, so that the square of the magnitude of the phasor is the optical power. For example, the optical power on the first input port is $|a_1|^2$, the second is $|a_2|^2$, and the power on the output waveguide is $|b|^2$. With this normalization, the optical power going into the device is

$$P_{in} = |a_1|^2 + |a_2|^2$$
Let the waveguides be made of materials with constant material parameters such as permittivity and permeability. If those material parameters are not time-varying and do not depend on the optical intensity, the device is called linear, and it can be modelled with the Maxwell equations, which are themselves linear equations.

Linear equations have the convenient property that the solution for one set of input optical amplitudes can be added to a different solution, and the sum is also a solution. In particular, if $b$ is calculated in the case in which $a_2$ is 0, the output, $b$, will be found to be a multiple of the input, owing to the linear nature of the solution:

$$b = c_1 a_1 \quad (2)$$

In the case $a_1 = 0$, the value of $b$ will also be proportional to the input, although the constant of proportionality could be different in the general case:

$$b = c_2 a_2 \quad (3)$$

If the solution is known in these two cases it is possible to determine the output when light is found on both inputs. Since the Maxwell equations are linear, the phasors are related by a linear equation also:

$$b = c_1 a_1 + c_2 a_2 \quad (4)$$

This coupler is symmetric, and therefore the coefficients $c_1$ and $c_2$ must be equal. Writing $c$ in place of both of these, we get

$$b = c (a_1 + a_2) \quad (5)$$

Considerations of conservation of energy can reveal constraints on possible values of $c$. (No, unfortunately, $c = 1$ is not a possibility!). The power leaving on the output waveguide has to be less than or equal to the sum of the input powers:

$$P_{out} = b\overline{b} = c\overline{c} |a_1 + a_2|^2 \leq P_{in} = |a_1|^2 + |a_2|^2 \quad (6)$$

This inequality establishes a constraint on the modulus of $c$:

$$|c| \leq \frac{\sqrt{|a_1|^2 + |a_2|^2}}{|a_1 + a_2|} \quad (7)$$

The above inequality must hold for any choice of $a_1$ and $a_2$. (There are no exceptions to the law of conservation of energy!). This inequality can be used to find the theoretical maximum of $|c|$, and thereby establish the specification for the most efficient coupler possible. The theoretical maximum of $|c|$ will be the minimum possible value of the right hand side of (7). Figure 2 shows a graphical representation of the quantities in inequality (7). The left hand side, $|c|$, needs to be smaller than any possible configuration in Figure 2.
Figure 2, Graphical representation of quantities in (7)

Once the size and direction of $a_1$ is determined, the ratio on the right hand side of (7) can be imagined by rotating the vector $a_2$ around the end of $a_1$. The numerator of (7) is fixed by the length of $a_1$ and $a_2$, and the denominator can vary by changing the direction of $a_2$. From these observations it is clear that the minimum is obtained when $a_1$ and $a_2$ are co-linear, since that maximizes the length of $a_1 + a_2$, as shown in Figure 3.

Figure 3, Configuration to get the minimum ratio

Once the phasors are in this configuration, the absolute minimum can be found from a linear search of the ratio of $a_1$ and $a_2$. Let

$$r = \frac{a_2}{a_1} \quad (8)$$

Then the right hand side of (7) is a function of the single variable, $r$. Let the value of the right hand side of (7) be written as $f(r)$, so that

$$f(r) = \frac{\sqrt{1 + r^2}}{1 + r} \quad (9)$$
Elementary calculus shows that there is a minimum at the point \( r = 1 \), and that the minimum is \( 1/\sqrt{2} \). Therefore the optimum coupler will be one in which the equality of (7) is satisfied, and

\[
b = \frac{1}{\sqrt{2}} (a_1 + a_2) \tag{10}
\]

Suppose that \( a_1 \) and \( a_2 \) have the same amplitude \( (r = 1) \) and phase, \( a_1 = a \) and \( a_2 = a \). Then for an optimum coupler we can apply the above to get

\[
b = \frac{1}{\sqrt{2}} (a_1 + a_2) = \sqrt{2} a \tag{11}
\]

and squaring both sides to see the power out:

\[
P_{\text{out}} = |b|^2 = 2|a|^2 = P_{\text{in}} \tag{12}
\]

The theory predicts that it will be possible to combine two optical powers and get all of the input power transferred to the output. On the other hand, the same theory predicts that if one of those inputs is turned off, the same coupler will experience loss on the remaining input. Let the input on port 2 be off so that \( a_2 = 0 \)

\[
P_{\text{out}} = |b|^2 = \left| \frac{1}{\sqrt{2}} a \right|^2 = \frac{1}{2} P_{\text{in}} \tag{13}
\]

When one of the inputs is turned off, the remaining input experiences a 3 dB loss, not seen when the other input was on! The analysis also points to the frustrating conclusion that even if the two inputs are of the same phase, but not equal power, there will still be losses!
Simulation by OptiBPM

To understand how the above conclusion could be possible, it helps to see a detailed simulation of an example. The design of Figure 1 has been entered into an OptiBPM project. The waveguides are of a low index contrast buried channel design, and they are brought together slowly over a distance of 800 µm as shown in Figure 4.

The horizontal axis is the propagation axis, Z. The vertical red line (corresponding to Z = 0) marks the beginning of the simulation by the Beam Propagation Method. The initial field in the plane marked by the red line can be anything, and in the first simulation it is taken to be the fundamental modes of the two input waveguides. The modes are initialized so that the optical phase is the same on both waveguides. OptiBPM will use these initial conditions to propagate forward (left to right) until the end of the simulation at Z = 800 µm. The result is shown in Figure 5.

In the OptiBPM simulation, the light in the two waveguides add together, and the amplitude increases accordingly. OptiBPM can calculate the power in the output waveguide. In the simulation of Figure 5, the total power at the input is normalized to 1, and the power at the output is calculated to be 0.9644. The insertion loss is only 0.16 dB, therefore this device functions as an efficient power combiner.
To understand what happens when one of the inputs is dark, the same simulation is repeated. However, this time the initial conditions are set such that there is light only in the lower input waveguide. The result is shown in Figure 6.

![Figure 6](image1.png)

**Figure 6, Input from lower waveguide only, upper waveguide dark**

In the simulation with input on one waveguide only, there appears to be a multimode coupling effect at the entrance of the combiner. The optical pattern is such that much of the optical power is lost to radiation. The power in the output waveguide is calculated to be 0.4822, exactly half the power coupled to the output waveguide as in Figure 5. When one waveguide is used for input, half the power is lost. The simulation by OptiBPM agrees with the theoretical prediction, and the effect is observed and can be understood as a wave interference effect. When \( a_1 = a_2 \), there is an interference between the two waves so as to make the total field pattern similar to the field pattern of the output waveguide, which permits efficient coupling to the output. However, if \( a_1 = 0 \), there is no such interference, and the optical field pattern is superimposed on only half the output mode pattern. Interference of waves can cause a reduction in amplitude, and that appears to be happening in this case.

OptiBPM can also investigate what happens when the inputs are equal but opposite, \( a_1 = -a_2 \), in which case (10) predicts there should be no power coupled. In Figure 7, the simulation is shown. The input appears the same as in Figure 5, but the phase is the opposite.

![Figure 7](image2.png)

**Figure 7, Two inputs with the same amplitude but opposite phase**

The light in the waveguides seem to avoid each other, as a result of destructive interference. OptiBPM calculates the power in the output waveguide to be -100 dB. It seems that, in optics, opposites do not attract!