Nonlinear Electrical Compensation for the Coherent Optical OFDM System

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***Abstract*—A main drawback of Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM) system is its sensitivity to fiber nonlinearity. Nonlinear electrical equalizer based on Volterra model has been demonstrated capable of compensating fiber nonlinear distortion in an OOK or PSK optical communication system. However, the implementation complexity of a Volterra model based electrical equalizer prohibits its deployment in a real-life CO-OFDM system. In this paper, we demonstrate that the number of kernels of a Volterra model based equalizer can be significantly reduced using the modified Gram-Schmidt method with reorthogonalization techniques. The resulting "sparse" Volterra model based electrical equalizer and the electrical equalizer based on the "full" Volterra model have comparable performance and can compensate intra-channel nonlinearity of a 16-QAM 100 Gbit/s CO-OFDM System.**

***Index Terms*—Equalizers, nonlinear distortion, nonlinear systems, orthogonal functions, optical fiber communication, OFDM, Volterra series.**

I. Introduction

COHERENT Optical OFDM (CO-OFDM) is considered an enabling technology of the next generation optical communication system since it possesses the merits of both a coherent system and an OFDM system [1]. As a coherent system, the CO-OFDM system maintains both signal amplitude and phase [2], thus increasing bandwidth utilization. The coherent optical communication system also makes full compensation of chromatic dispersion after optical/electrical conversion possible. The OFDM modulation scheme leads to a high spectral efficiency owing to its partially overlapping subcarriers [1]. Moreover, the cyclic prefix code of the CO-OFDM system makes the system more resistant to inter-symbol interference caused by chromatic dispersion and polarization mode dispersion (PMD) [1], [3].

One major concern people have about the CO-OFDM system is its vulnerability to fiber nonlinear effects such as self-phase modulation (SPM) and cross-phase modulation (XPM). Both SPM and XPM are caused by the optical signal intensity fluctuation [4]. Since the OFDM system has a high peak-to average power ratio (PAPR) [5], a CO-OFDM system has more severe SPM and XPM compared with traditional optical communication systems. Because the OFDM is a multi-carrier modulation scheme, the four-wave mixing (FWM) among subcarriers within one channel introduces additional distortion [6]. As a result, nonlinearity compensation is a crucial component of the CO-OFDM system. In this paper, we concentrate on intra-channel nonlinearity of the CO-OFDM system caused by SPM and FWM among subcarriers and present a Volterra model based nonlinear signal processing scheme to compensate the intra-channel nonlinearity.

The Volterra model is a widely used nonlinear signal processing tool [7]. It has been used to model the optical communication system nonlinearity [8]–[12]. The Volterra model is also used to mitigate nonlinearity effects in optical communication systems [13] and design equalizers for optical systems with OOK and PSK modulation [6], [11], [14]–[16].

The biggest disadvantage of a Volterra model based nonlinear compensator is its complexity. A considerable amount of Volterra model coefficients is usually required to model a nonlinear system. Consequently, it may not be feasible to apply a Volterra model based compensator in real-time signal processing applications [17]. One possible solution is to identify the most significant coefficients of a Volterra model and delete all of the insignificant coefficients from the Volterra model [17], [18]. The resulting Volterra model is referred to as a sparse Volterra model by some researchers [19].

In this paper, the nonlinear effect of a 16-QAM, 100 Gbit/s CO-OFDM system is investigated and electrical equalizers based on the linear model, conventional Volterra model, and sparse Volterra model are designed and tested in simulations. The significant coefficients of the Volterra model are identified using reorthogonalization techniques [19]. It is shown that nonlinear equalizers clearly outperform linear equalizers and equalizers based on full Volterra model and sparse Volterra model have comparable performances. To the best of authors' knowledge, no study on sparse Volterra model based nonlinear compensators for the CO-OFDM system has been conducted and the research results presented in this paper can lead to the rapid development and deployment of CO-OFDM systems for the next generation optical communication systems.

The rest of this paper is organized as follows: the Volterra model is introduced in the second section, the CO-OFDM system simulation diagram used in our study is described in the third section, the simulation results and discussions are presented in the fourth section, and the fifth section concludes this paper.

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II. Volterra Model

The Volterra series can be considered as a Taylor series with memory and the input-output relation of a discrete-time Volterra model is given below

$$y(n) = \sum_{i=0}^{\infty} h_1(i)x(n-i) + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} h_2(i,j)x(n-i)x(n-j) + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} h_3(i,j,k)x(n-i)x(n-j)x(n-k) + \cdots \quad (1)$$

where $x[\cdot]$ is the input signal, $y[\cdot]$ is the output signal, and $h_i[\cdot]$ is the ith-order kernels of the Volterra model.

Because of the bandpass nature of the communication channel [20], the input and output signals of communication system are often represented by signals' complex envelopes. When the Volterra model is used to model the complex envelope input-output relations of a bandpass system, the even-order Volterra kernels is ignored because they do not generate in-band signal. A discrete causal third-order bandpass Volterra model with finite memory length can be represented using the following equation [21]:

$$y(n) = \sum_{i=0}^{N} h_1(i)x(n-i) + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N} h_3(i,j,k)x(n-i)x(n-j)x^*(n-k) + e(n) \quad (2)$$

where N is the memory length, $(\ )^*$ denotes the complex conjugate, $x(n)$ and $y(n)$ are the input and output signal complex envelopes, $e(n)$ is modeling error, $h_1(i)$ and $h_3(i,j,k)$ are the linear and cubic Volterra kernels respectively.

Usually, to model a highly nonlinear system, a Volterra model with higher order or longer memory length is desirable. However, an increase in memory length or system order will increase the number of kernels exponentially and result in complicated calculations. Since not every Volterra kernel has equal importance, the coefficients determination of a high-order Volterra model with long memory length is more difficult due to a large number of "unimportant" kernels [19]. As a result, it is necessary to identify and remove the unimportant kernels to simplify the Volterra system structure in practical applications. We apply the orthogonal search approach to reduce the number of Volterra kernels [17], [18].

To use the orthogonal search approach to reduce the number of Volterra kernels, the filter coefficients are updated and searched in the orthogonal domain, making the search process more straightforward. The detail procedure can be described as following. The (2) can be rewritten as [17]

$$y(n) = \sum_{i=1}^{L} w_i u_i(n) + e(n) \quad (3)$$

where L is the total number of Volterra kernels, u_i is the linear or cubic input terms, and w_i is the corresponding Volterra kernels.

For K input/output samples, we can write the input-output equations in vector forms

$$Y = \bar{Y} + E = \sum_{i=1}^{L} w_i U_i + E \quad (4)$$

where

$$Y = [y(1)\, y(2)\, \ldots\, y(K)]^T,$$
$$U_i = [u_i(1)\, u_i(2)\, \ldots\, u_i(K)]^T$$

and

$$E = [e(1)\, e(2)\, \ldots\, e(K)]^T$$

We would like to find the Volterra kernels which have more significant contributions to the output than others. This task can be easily accomplished in an orthogonal domain. In the orthogonal domain, (4) is transformed to

$$Y = \bar{Y} + E = \sum_{i=1}^{L} \mathrm{v}_i Q_i + E \quad (5)$$

where Q is the orthogonalized matrix of vectors U. v_i is the orthogonal Volterra coefficients. Since matrix Q is orthogonal, the coefficients v_i can be determined by [17]

$$\mathrm{v}_i = \frac{Y^T Q_i}{Q_i^T Q_i}. \quad (6)$$

The model's normalized mean square error (NMSE) can be represented as:

$$\frac{E^T E}{Y^T Y} = \frac{\left(Y - \sum_{i=1}^{L} \mathrm{v}_i Q_i\right)^T \left(Y - \sum_{i=1}^{L} \mathrm{v}_i Q_i\right)}{Y^T Y} = 1 - \frac{\sum_{i=1}^{L} \mathrm{v}_i^2 \mathrm{Q}_i^T Q_i}{Y^T Y} = 1 - \sum_{i=1}^{L} D_i \quad (7)$$

$$D_i = \frac{\mathrm{v}_i^2 \mathrm{Q}_i^T Q_i}{Y^T Y} \quad (8)$$

The problem is now simplified to identify the most significant v_i among the L coefficients which contributes to a large D_i in (8). The modified Gram-Schmidt orthogonal decomposition method can be used to generate orthogonalized matrix, Q [17], [18], [22]. The pseudo code of the modified Gram-Schmidt method is given as follows.

```
  Q = U;
for j = 1 : L
    a = norm(Q(:, j))
    for i = 1 : j − 1
        R(i, j) = Q(:, i)' ∗ Q(:, j)
  prob: Q(:, j) = Q(:, j) − R(i, j) ∗ Q(:, i)
    end for
    b = norm(Q(:, j))
    R(j, j) = norm(Q(:, j))
    Q(:, j) = Q(:, j)/R(j, j)
end for
```

One problem associated with the modified Gram Schmidt method is that Q may lose orthogonality due to the intensive cancellation happened in the line labeled *prob* [23]. One way to detect the cancellation is that if $||b||_2 \leq \varepsilon||a||_2$ (ε is a value determines if the reorthogonalization needs to be performed), a reorthogonalization is needed [23], and the pseudo code above is then modified as [23].

```
  Q = U;
for j = 1 : L
    a = norm(Q(:, j))
  re_org: for i = 1 : j − 1
            s = Q(:, i)' ∗ Q(:, j)
            R(i, j) = R(i, j) + s
            Q(:, j) = Q(:, j) − s ∗ Q(:, i)
        end for
    b = norm(Q(:, j))
    if b < εa then (a = b) and (goto re_org)
    R(j, j) = b;
    Q(:, j) = Q(:, j)/R(j, j)
end for
```

When the input matrix is rank deficient, some vectors are linearly dependent. Those linear dependent vectors can be easily identified by evaluating the value of b. If the value b has very small value, then the corresponding R and the related Q will be set to zero. Suppose the linear dependent vector indexes are k, and linear independent vector indexes are j, (5) would become

$$Y = \bar{Y} + E = \sum_j \mathrm{v}_j Q_j + \sum_k \mathrm{v}_k Q_k + E = \sum_j \mathrm{v}_j Q_j + E \tag{9}$$

Assume that M is the rank of the input matrix ($M \leq L$), the number of "valid" coefficients would be M, with the rest of coefficients set equal to zero. The kernels of the sparse Volterra model are constructed from these M coefficients. Since $Q_m^T Q_n = 0$ ($m \neq n$) and $Q_m^T Q_m = 1$, we can derive the following equations based on (6) and (8):

$$\mathrm{v}_j = Y^T Q_j \tag{10}$$

$$D_j = \frac{\mathrm{v}_j^2}{Y^T Y}. \tag{11}$$

Each time, the coefficient v_x, which generates the biggest D_x, is selected and NMSE = NMSE $- D_x$. The search process ends once the NMSE meets the pre-determined value.

To construct sparse Volterra models, the coefficient matrix, V, will still have size of L with all the unimportant coefficients set to zero. The back substitution method is then used to determine the sparse Volterra model kernels [22]. The resulting sparse Volterra model will have less non-zero kernels than the "full" Volterra model.

III. System Description and Modeling

In our study, the CO-OFDM system is simulated by a commercial fiber optics system simulation tool, OptiSystem™. It has been used by many researchers to simulate the fiber nonlinearity and dispersion effects in optical communication systems [24], [25]. Our simulation setting takes most key optical communication system/component parameters into account including fiber nonlinearity, noise, dispersion, and PMD, etc. For the sake of simplicity, some effects such as the laser frequency drifting, and filter bandwidth drifting are ignored. Although these problems might be encountered in the real world transmission, their impacts on the system performance usually can be reduced by using more reliable components or increasing channel spacing. The CO-OFDM simulation configuration is illustrated in Fig. 1.

The data transmission bit rate is 100 Gbit/s. On the transmitter side, a bit stream is generated using a pseudo random binary sequence generator, and the data is mapped by a 16-QAM encoder. The information stream is further parsed into 128 low speed parallel data subcarriers and processed by the IFFT processor. Cyclic prefix is added to ensure a correct data recovery. The 25 Gbaud rate OFDM in-phase and quadrature parts then pass the lowpass filter. The Mach–Zehnder modulator is used to convert electrical signals to optical signals. The laser line width is set at 1 MHz, with adjustable launch power. The frequency of the carrier wave is set at 193.1 THz. The optical channel consists of 10 spans of 80 km standard single mode fiber (SSMF), with attenuation $= 0.2$ dB/km, dispersion $= 16$ ps/nm/km and nonlinearity coefficient $= 2.09$ /w/km. Fiber dispersion is fully compensated by the dispersion compensation fiber (DCF) in each span which has 0.6 dB/km attenuation, -80 ps/nm/km dispersion and 6.4/w/km nonlinearity coefficient. Both the SSMF and DCF span loss is balanced by a 4 dB noise figure optical amplifier in each loop. Optical Signal to Noise Ratio (OSNR) is measured at the end of transmission to evaluate the system performance.

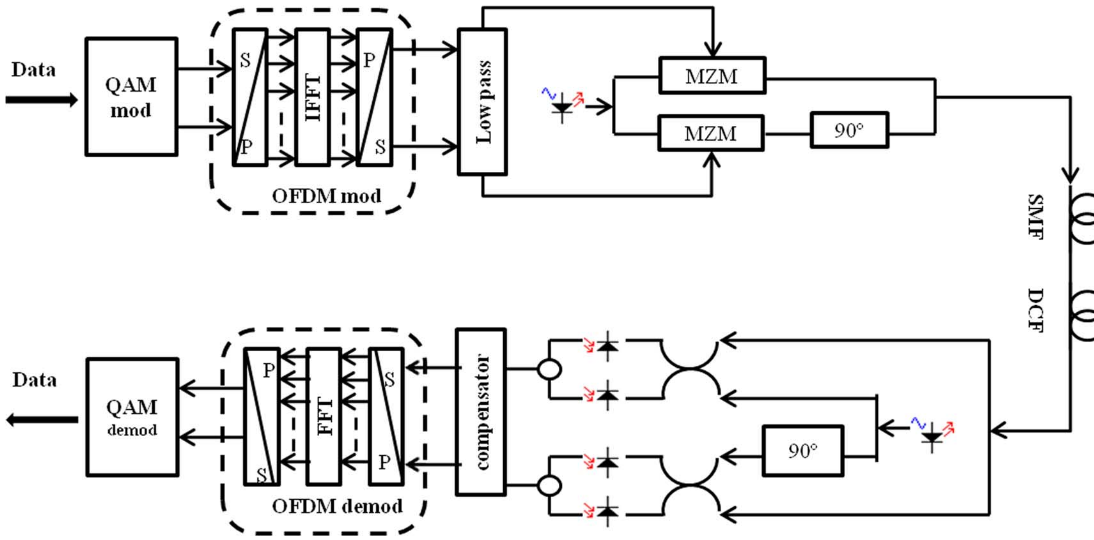

Fig. 1. Block diagram of CO-OFDM system.

Amplified spontaneous emission (ASE) noise is reduced by an optical filter at the receiver. The local oscillator (LO) laser is assumed to be perfectly aligned with power set at -2 dBm and line width equals to 1 MHz. The I/Q components of the OFDM signal is recovered by a 2×4 90 degree optical hybrid and two pairs of photo-detectors. Photo-detector noise, such as thermal noise, shot noise, dark current and ASE noise are included in the simulation. The converted OFDM RF signal is demodulated using FFT processor and the guarding interval is removed. The obtained signals are fed into a QAM decoder. Transmission bits are collected and bit error ratio (BER) is calculated and compared at the end of the receiver.

In a WDM setting, 5 channels of 25 Gbaud 16-QAM OFDM signals are transmitted. The carrier wave frequencies are set from 192.9 THz to 193.3 THz, with 100 GHz channel spacing. The transmission length is reduced to 9×80 km. Only the central channel is evaluated in the WDM simulation.

The equalization is realized by a third-order Volterra electrical equalizer, with memory length of two ($N = 2$ in (2)). The number of Volterra kernels is thirty. A training sequence is sent to determine the full Volterra electrical equalizer coefficients using the recursive least square (RLS) methods. The resulting equalizer is used to compensate different sequences (total number of bits: 2^{16}). To determine the sparse Volterra equalizer, a training sequence is sent out to identify the most significant terms of the full Volterra equalizer using the reorthogonalization method. These sparse Volterra equalizer taps can be updated using RLS algorithm for compensation.

IV. Results and Discussions

For comparison purpose, the adaptive linear equalizer is also included in the simulation to evaluate the performance of nonlinear equalizers. The received signal constellation diagram after 800 km fiber transmission, with 2 dBm launch power is shown in Fig. 2. Due to SPM, ASE noise and photodetector noise, the constellation diagram has become scattered and has phase and amplitude distortions. The linear kernels account for the attenuation and the dispersion effect of fiber, while the third order kernels can account for the interaction between ASE noise and signal and nonlinear distortion [13]. Since linear equalizer has no nonlinear terms, its capability of removing the phase noise introduced by fiber nonlinearity is restricted. As shown in the constellation diagram, there is no doubt that nonlinear equalizers outperform the linear equalizer. The sparse Volterra equalizer with only 8 kernels and full Volterra equalizer with 30 kernels have comparable performance as shown in Fig. 2.

The Monte Carlo simulations are conducted to evaluate the equalizer effectiveness on the OFDM system after 800 km of transmission. The resulting BER result is shown in Fig. 3. At low launch powers, the OFDM system with or without compensation have similar BER, and full Volterra equalizer appears to have similar performance as the linear equalizer. The reasons are that, under low input power level, the fiber amplifier can be modeled as a linear filter [13], the linear dispersion dominates [10], and the fiber nonlinearity effect is weak. A low OSNR at low launch power also limits the performance of nonlinear filter. When launch power increases, the system BER decreases at the first and then increases when launch power is larger than the "optimal" launch power. As shown in Fig. 3, OFDM systems with (or without) different equalizers have different "optimal" launch power and BER values. The OFDM system with the nonlinear equalizer can take higher launch power and reach lower BER.

The BER increase under high launch power is caused by a larger SPM and ASE noise. Peddanarappagari and Brandt-Pearce have shown that at higher power, for a fixed input pulse width, the detector output pulsewidth increases with higher peak pulse power and makes the system more sensitive to nonlinearity distortion [9]. As a result, the low order nonlinear equalizer with a short memory span is not able to fully compensate the nonlinear channel. Ideally, a better result can be obtained by increasing the order and memory length of the Volterra equalizer. However, the resulting increased equalizer implementation complexity might not be acceptable.

We repeat the Monte Carlo simulation to calculate the BER of OFDM systems with full Volterra equalizer and sparse Volterra

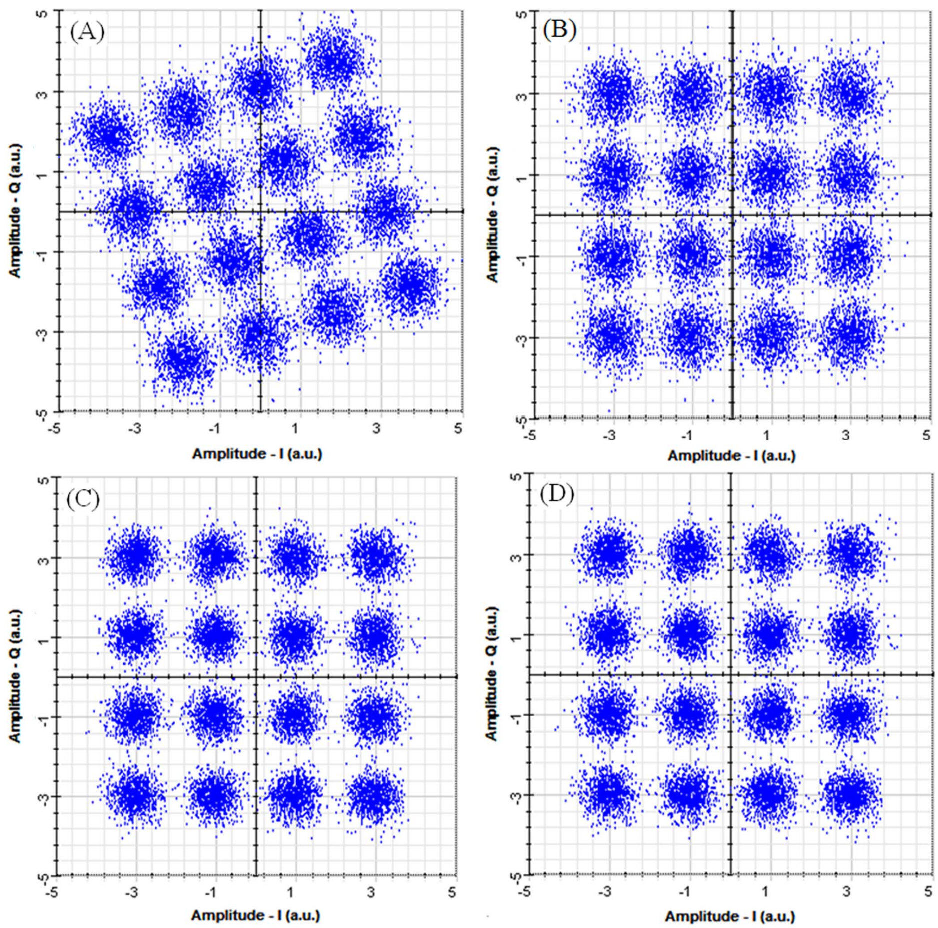

Fig. 2. Constellation of 16-QAM CO-OFDM system. (A) w/o equalizer, (B) with linear equalizer (C) with full Volterra equalizer (D) with sparse Volterra equalizer.

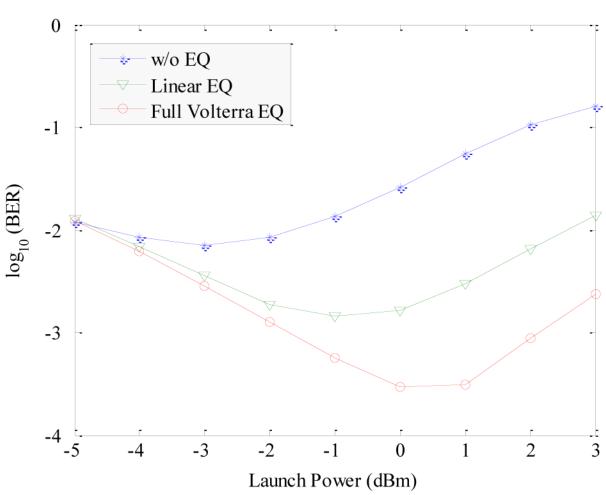

Fig. 3. BER of 16-QAM CO-OFDM systems w/o compensation and with linear/nonlinear compensation as a function of launch power.

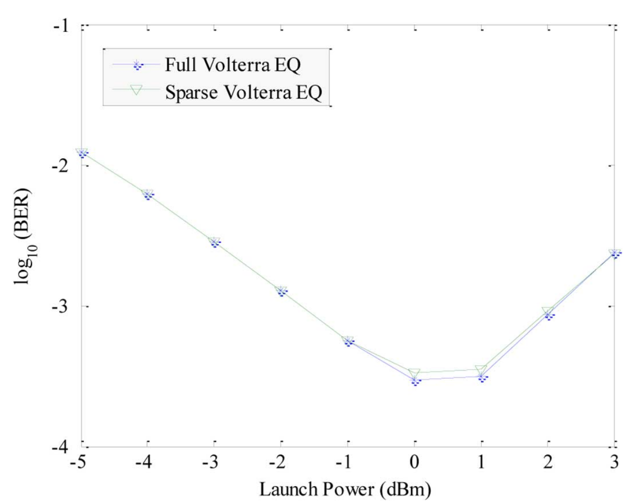

Fig. 4. BER of 16-QAM CO-OFDM systems with full or sparse Volterra equalizers as a function of launch power.

equalizer and the resulting BER curves are shown in Fig. 4. As indicated in Fig. 4, sparse Volterra and full Volterra equalizer have similar performances although the sparse Volterra equalizer has far less coefficients.

Fig. 5 shows the BERs of OFDM systems with no compensation and with linear, full Volterra, and sparse Volterra compensation at different OSNR under 0 dBm launch power. Additional ASE noise is added at the end of transmission to set different OSNR values. It is not surprising that with the increase of OSNR, the system would have a better performance. The outperformance of nonlinear compensators becomes more evident with the increase of OSNR, since the signal becomes less distorted and the estimation becomes more accurate. The sparse

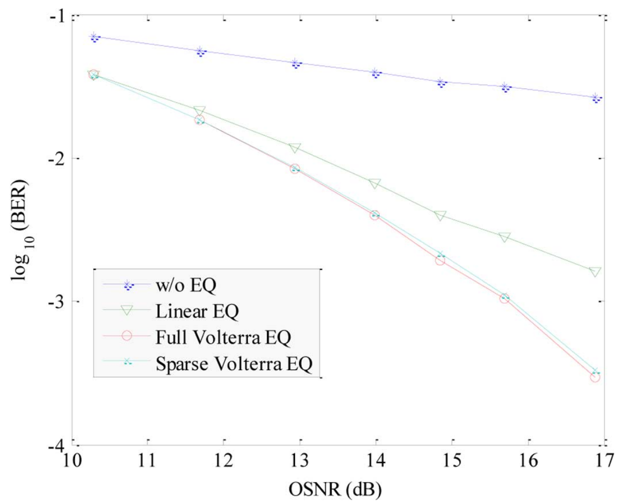

Fig. 5. BER of 16-QAM CO-OFDM systems w/o compensation and with linear/nonlinear equalization as a function of OSNR (fixed launch power).

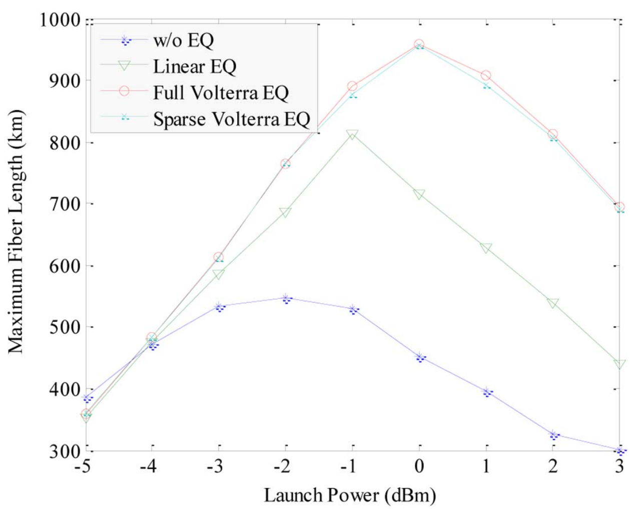

Fig. 6. 16-QAM CO-OFDM systems' maximum transmission distances at 10^{-3} BER versus launch power.

Volterra and full Volterra equalizers have similar performance as shown in Fig. 5 as well.

Simulations are also conducted to investigate reach limits of different OFDM systems under different launch power to guarantee a 10^{-3} BER and the results are included in Fig. 6. At lower launch power, the OFDM system without compensation can reach similar distance as the OFDM system with linear/nonlinear compensator. The maximum reach of the OFDM system occurs at the same launch power in Fig. 3 which gives the lowest BER value. With the nonlinear equalization, the OFDM system can approach 1000 km transmission distance at 0 dBm launch power. As shown in Fig. 6, the OFDM system with nonlinear compensation can take larger launch power and reach longer distance. At higher launch power, the linear and nonlinear equalizers can only support less transmission length because of high nonlinearity.

The BER versus (launch power/per channel) simulation is conducted for 5-channel WDM CO-OFDM systems with or without compensation. The transmission distance is reduced

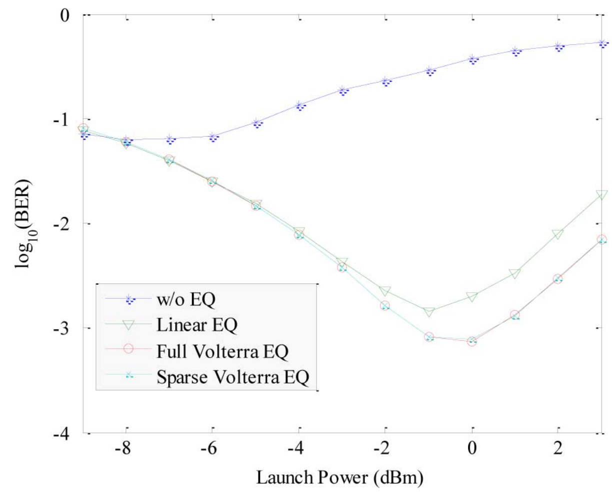

Fig. 7. BER of 5 channel WDM 16-QAM CO-OFDM systems w/o compensation and with linear/nonlinear compensation as a function of launch power.

to 9×80 km instead of 10×80 km. The BER of the center wavelength is calculated and simulation results are shown in Fig. 7. Comparing Fig. 7 with Fig. 3, we can see that, even with a shorter transmission distance, the BER performance of the WDM CO-OFDM system is still worse than the single channel CO-OFDM system. The reason is that, in the WDM system, XPM leads to amplitude distortion and timing jitters [26]. Furthermore, XPM is proportional to the optical power [27]. With the increase of total launch power, the BER of WDM CO-OFDM system without compensation deteriorates. The WDM CO-OFDM system with linear or nonlinear compensation show similar tendency as shown in Fig. 3. The nonlinear equalizer still has a better performance, indicating that the Volterra equalizer can be used to compensate intra-channel nonlinearity in a WDM system. The sparse Volterra equalizer also functions well with much less equalizer taps.

V. Conclusion

This paper presents the investigation on system nonlinearity of single channel and WDM 100 Gbit/s 16-QAM CO-OFDM systems and its compensation. The Volterra model based electrical equalizer has been shown capable of compensating intra-channel nonlinearity of the CO-OFDM system. A simple method is proposed to obtain a sparse Volterra equalizer using the modified Gram-Schmidt method with reorthogonalization. It is shown that, with much less coefficients, the sparse Volterra equalizer can deliver performance similar to the full Volterra equalizer. The results presented in this paper can ease research community's concern about the Volterra equalizer and lead to the successful development of a CO-OFDM system.

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